Discussion

Comments on “A class of proportional-integral sliding mode control with application to active suspension system”∗

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Received 30 May 2005; accepted 7 October 2006
Available online 11 January 2007

Abstract

In this note, we first show that Theorem 2 in the above paper is incorrect. Then, we also show that Theorem 1 is not enough to support the claim in the above paper that the proportional-integral sliding mode control method is more robust than the linear quadratic regulator method when applying on the active suspension system with mismatched uncertainties.

Keywords: Automotive control; Quarter-car suspension; Sliding mode control; Robust control; Mismatched uncertainties

It is interesting and inspiring that the above paper [1] proposed a proportional-integral sliding mode controller (PISMC) to achieve more robust performance of an active suspension system, which suffers mismatched uncertainties resulting from the road disturbances.

The considered system in the paper [1] is modeled as

\[ \dot{x}(t) = Ax(t) + Bu(t) + f(t), \]  

where \( x(t) \in \mathbb{R}^n \) is the state vector, \( u(t) \in \mathbb{R} \) is the control input and the continuous function \( f(t) \) represents the mismatched uncertainties. The introduced assumptions are:

(A1) There exists a known positive constant \( \beta \) such that \( \| f(t) \| \leq \beta \).  

(A2) The pair \( (A, B) \) is controllable and the input matrix \( B \) has full rank.

The utilized PI sliding surface is defined as

\[ \sigma(t) = Cx(t) - \int_0^t (CA + CBK)x(\tau) \, d\tau, \]

where \( C \in \mathbb{R}^{m \times n} \) and \( K \in \mathbb{R}^{m \times n} \) are constant matrices to be designed. Theorem 2 in the paper [1] is incorrect to show the hitting condition, the inequality (12) in [1], of the sliding surface (2), because the sign of the second term of the last line of formula (13) in [1] is contrary. This error means

\[ -\sigma^T(t)(CA + CBK)x(t) \]

\[ \leq -\|C\|\|A + BK\|\|x(t)\|\|\sigma(t)\|, \]

which does not hold in general. Fortunately, there are many extant methods to design the control law to satisfy the hitting condition of sliding surface.

**Theorem 1.** Set \( k = \beta\|C\| + \eta \) with \( \eta \) an arbitrary positive scalar. Under the following control law

\[ u(t) = (CB)^{-1} \left( Kx(t) - \frac{k\sigma(t)}{\|\sigma(t)\| + \delta} \right), \]

with a small positive scalar \( \delta \), the trajectory of the system (1) will in finite time enter and remain in the neighboring domain

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∗This work was supported by the National Outstanding Youth Science Foundation of China (60025308) and Priority supported financially by “the New Century 151 Talent Project” of Zhejiang Province.

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∥ · ∥ denotes the standard Euclidean norm throughout this note.
of the sliding surface
\[ \Omega = \left\{ x(t) : \| \sigma(t) \| \leq \frac{\beta \| C \| + \eta_1 \delta}{\eta - \eta_1} \right\}, \quad (5) \]
where \( 0 < \eta_1 < \eta \). When \( \delta \to 0 \), \( \Omega \to \sigma(t) = 0 \).

**Proof.** Consider the Lyapunov function \( V = \sigma^2 \). Its derivative along system (1) is
\[ \dot{V} = 2\sigma(C Bu + C f - CB K x). \quad (6) \]
Applying the control law (4) yields
\[ \dot{V} \leq 2\sigma \left( C f - k \frac{\sigma}{\| \sigma \| + \delta} \right) \leq 2\sigma \left( \beta \| C \| - k \frac{\sigma}{\| \sigma \| + \delta} \right). \quad (7) \]
Further, we have
\[ \dot{V} \leq -2\eta_1 \| \sigma \| \quad \forall x(t) \notin \Omega, \quad (8) \]
which means that the trajectory of the system will sustain in the neighboring domain of sliding surface \( \Omega \) in finite time. Moreover, if \( \delta = 0 \), the trajectory will reach the sliding surface \( \sigma(t) = 0 \) in finite time and subsequently remain on it. \( \square \)

In the paper [1], Theorem 1 shows that the system trajectory will ultimately remain in a closed ball \( B(\eta) \), centered at \( x = 0 \) with radius \( \eta = 2\beta_1 \| P \| /\| \sigma \| \min(Q) \), where \( \beta_1 = \| I_n - B(C B)^{-1} C \| \beta \), symmetric positive definite matrices \( P \) and \( Q \) satisfy
\[ P(A + BK) + (A + BK)^T P = Q. \quad (9) \]
In fact, the bounded stability can also be achieved by the general state feedback \( u(t) = K x(t) \), where the closed-loop system is
\[ \dot{x}(t) = (A + BK)x(t) + f(t). \quad (10) \]
As a well-known result, the trajectory of the closed-loop system (10) will ultimately sustain in the closed ball \( B(\delta) \), centered at \( x = 0 \) with radius \( \delta = 2\beta_1 \| P \| /\| \sigma \| \min(Q) \), where \( P \) and \( Q \) are the same as (9). It can be seen that
\[ B(\delta) \subseteq B(\eta) \quad (11) \]
because of \( \| I_n - B(C B)^{-1} C \| \geq 1 \), which comes from the definition of Euclidean norm and
\[ (I_n - B(C B)^{-1} C)C^\perp = C^\perp, \quad (12) \]
where \( C^\perp \in \mathbb{R}^{n \times (n-m)} \) is the orthogonal complement matrix of \( C \). The relation (11) shows that if the uncertainty is mismatched, i.e., \( (I_n - B(C B)^{-1} C) f(t) \neq 0 \), the robustness achieved by the PISM C is not stronger than that by the general static feedback control. For example, consider a planar system with \( B = [0, 1]^T \), \( f(t) = [1, 0]^T r(t) \) and \( C = [c_1, c_2] \), where \( r(t) \) is a scalar function satisfying \( |r(t)| \leq \beta \), \( c_1 \) and \( c_2 \) are positive scalars with \( c_2 \neq 0 \). Applying the general state feedback, the closed-loop system dynamics is
\[ \dot{x}(t) = (A + BK)x(t) + \left[ \begin{array}{c} 1 \\ 0 \end{array} \right] r(t). \quad (13) \]
However, using the PISM C method in the paper [1], the equivalent dynamics of the closed-loop system in sliding mode is
\[ \dot{x}(t) = (A + BK)x(t) + \left[ \begin{array}{c} 1 \\ -c_2^{-1} c_1 \end{array} \right] r(t). \quad (14) \]
By the comparison between (14) and (13), we can see that PISM C is more sensitive to the mismatched uncertain than the general state feedback control under the same feedback gain \( K \). In particular, if \( c_2^{-1} c_1 \gg 1 \), then the influence of the mismatched uncertainties will be not reduced but enlarged by PISM C.

In view of the above, we conclude that Theorem 1 in the commented paper [1] is not able to support the property of the proposed PISM C that it is more robust than the general control approach. Albeit the simulation on the active suspension system in the paper [1] shows its argument that the PISM C is more robust to the mismatched uncertainty than the LQR (Linear Quadratic Regulator) control approach, we think it is not strict and does not complement the theory and should be confirmed further.

**Reference**